题: find the quadratic form

$q(x.y.z) = x^2 + 2y^2 + z^2 + 4xy + 6xz + 4yz$

by each method

a) By square completion

b) By characteristic roots of the representation matrix

解: a) $q(x.y.z) = x^2+2y^2+z^2+4xy+6xz+4yz$

$= x^2 + 2(2y+3z)x + 2y^2 + z^2 + 4yz$

$= (x+2y+3z)^2 - (2y+3z)^2 + 2y^2 + z^2 + 4yz$

$= (x+2y+3z)^2 - 4y^2 - 12yz - 9z^2 + 2y^2 + z^2 + 4yz$

$= (x+2y+3z)^2 - 2y^2 - 8yz - 8z^2$

$= (x+2y+3z)^2 - 2(y^2 + 2(2z)) - 8z^2$

$= (x+2y+3z)^2 - 2[(y+2z)^2 - 4z^2] - 8z^2$

$= (x+2y+3z)^2 - 2(y+2z)^2$

So the signture of $q(x.y.z)$ is $(1\ 1.1)$,

b) $q(x,y,z) = (x.y.z)A\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 2 & 1 \end{pmatrix}$, $|A - \lambda I| = \begin{vmatrix} 1-\lambda & 2 & 3 \\ 2 & 2-\lambda & 2 \\ 3 & 2 & 1-\lambda \end{vmatrix}$

$= \begin{vmatrix} -2-\lambda & 0 & 2+\lambda \\ 2 & 2-\lambda & 2 \\ 3 & 2 & 1-\lambda \end{vmatrix} = -(2+\lambda)\begin{vmatrix} 2-\lambda & 2 \\ 2 & 1-\lambda \end{vmatrix} + (2+\lambda)\begin{vmatrix} 2 & 2-\lambda \\ 3 & 2 \end{vmatrix}$

$= -(2+\lambda)((2-\lambda)(1-\lambda)-4) + (2+\lambda)(4-3(2-\lambda))$

$= -(2+\lambda)(\lambda^2-3\lambda-2) + (2+\lambda)(3\lambda-2)$

$= (2+\lambda)(-\lambda^2+3\lambda+2+3\lambda-2) = (2+\lambda)(-\lambda^2+6\lambda)$

$= -\lambda(\lambda+2)(\lambda-6) = 0$

$\lambda_1 = 6$, $\lambda_2 = -2$, $\lambda_3 = 0$

So the signture of the $q(x.y.z)$ is $(1,1,1)$

(正,负,0的个数)