

题: 求曲线 $\begin{cases} x^2+y^2+z^2-3x=0 \\ 2x-3y+5z-4=0 \end{cases}$
在点 $(1, 1, 1)$ 处的切线及法平面方程

解: $\begin{cases} 2x + 2y \frac{\partial y}{\partial x} + 2z \frac{\partial z}{\partial x} - 3 = 0 \\ 2 - 3 \frac{\partial y}{\partial x} + 5 \frac{\partial z}{\partial x} = 0 \end{cases}$
 $\begin{cases} 2y \frac{\partial y}{\partial x} + 2z \frac{\partial z}{\partial x} = 3 - 2x \\ -3 \frac{\partial y}{\partial x} + 5 \frac{\partial z}{\partial x} = -2 \end{cases}$

$$\frac{\partial y}{\partial x} = \frac{\begin{vmatrix} 3-2x & 2z \\ -2 & 5 \end{vmatrix}}{\begin{vmatrix} 2y & 2z \\ -3 & 5 \end{vmatrix}} = \frac{15 - 10x + 4z}{10y + 6z}$$

$$\frac{\partial z}{\partial x} = \frac{\begin{vmatrix} 2y & 3-2x \\ -3 & -2 \end{vmatrix}}{\begin{vmatrix} 2y & 2z \\ -3 & 5 \end{vmatrix}} = \frac{-4y + 9 - 6x}{10y + 6z}$$

$$\left. \frac{\partial z}{\partial x} \right|_{(1,1,1)} = \frac{-4 + 9 - 6}{10 + 6} = -\frac{1}{16}$$

$$\left. \frac{\partial y}{\partial x} \right|_{(1,1,1)} = \frac{15 - 10 + 4}{10 + 6} = \frac{9}{16}$$

曲线切向量为 $T = (1, \frac{9}{16}, -\frac{1}{16})$ 或 $(16, 9, -1)$

故所求切线方程为:

$$\frac{x-1}{1} = \frac{y-1}{\frac{9}{16}} = \frac{z-1}{-\frac{1}{16}}$$

化简为 $\frac{x-1}{16} = \frac{y-1}{9} = \frac{z-1}{-1}$

法平面方程为:

$$16(x-1) + 9(y-1) - (z-1) = 0$$

$$16x + 9y - z - 24 = 0$$