题关键词：分部积分，三解代换，有理函数裂项积分

题：计算不定积分 $\int \frac{x\ln(x+\sqrt{1+x^2})}{(1-x^2)^2}dx$

解：分部积分公式∫udv=uv-∫vdu，令 u=ln(x+√(1+x²))，du=1/√(1+x²)dx，令 dv=x/(1-x²)²dx， v=1/2(1-x²)；

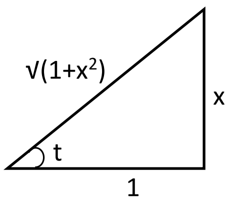
$$\int \frac{x\ln(x+\sqrt{1+x^2})}{(1-x^2)^2}dx=\int \ln(x+\sqrt{1+x^2})d(\frac{1}{2(1-x^2)})=\frac{\ln(x+\sqrt{1+x^2})}{2(1-x^2)}-\frac{1}{2}\int \frac{1}{(1-x^2)\sqrt{1+x^2}}dx$$

三角代换，令 x=tant, dx=sec²tdt ， 1+x²=1+tan²t=sec²t；第二次换元，令 y=sint

则 $\int \frac{1}{(1-x^2)\sqrt{1+x^2}}dx=\int \frac{1}{(1-\tan^2 t)\sqrt{1+\tan^2 t}}\sec^2 tdt=\int \frac{\cos t}{\cos^2 t-\sin^2 t}dt=\int \frac{1}{1-2\sin^2 t}d(\sin t)$

$$=\int \frac{1}{1-2y^2}dy=\int \frac{1}{(1-\sqrt{2}y)(1+\sqrt{2}y)}dy=\frac{1}{2}\int \frac{1}{1+\sqrt{2}y}+\frac{1}{1-\sqrt{2}y}dy=\frac{1}{2\sqrt{2}}\ln\left|\frac{1+\sqrt{2}y}{1-\sqrt{2}y}\right|+c$$

$$=\frac{1}{2\sqrt{2}}\ln\left|\frac{1+\sqrt{2}\sin t}{1-\sqrt{2}\sin t}\right|+c=\frac{1}{2\sqrt{2}}\ln\left|\frac{1+\sqrt{2}\frac{x}{\sqrt{1+x^2}}}{1-\sqrt{2}\frac{x}{\sqrt{1+x^2}}}\right|+c$$



$$=\frac{1}{2\sqrt{2}}\ln\left|\frac{\sqrt{1+x^2}+\sqrt{2}x}{\sqrt{1+x^2}-\sqrt{2}x}\right|+C$$

根据三角函数定义
Sint=x/√(1+x²)

$$\therefore \int \frac{x\ln(x+\sqrt{1+x^2})}{(1-x^2)^2}dx=\frac{\ln(x+\sqrt{1+x^2})}{2(1-x^2)}-\frac{1}{4\sqrt{2}}\ln\left|\frac{\sqrt{1+x^2}+\sqrt{2}x}{\sqrt{1+x^2}-\sqrt{2}x}\right|+C$$