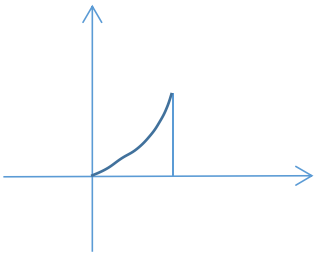
Problem.

Let $f(x,y)=y^2-2y-3yx^2$. (a) Find the critical points of f(x,y). (b) Find the absolute maximum and minimum values of f on the closed and bounded set enclosed by $x=\sqrt{y}$, x=1 and y=0 .

Solution

(a) $f_x=6x=0$, $f_y=2y-2-3x^2=0$, solve x=0 or y=1, the critical point of f(x,y) is (0,1)

(b) 1. the point (0,1) isn't in the set.



2. Find the extreme values on the boundary of the set

(1) Along y=0 f(x,0)=0

(2) Along x=1 $f(1,y)=y^2-5y$, $0<=y<=1$, $f_y(1,y)=2y-5=0$, $y=5/2>1$, f(1,1)=1-5=-4, f(1,0)=0 , so maxf(1,y)=-4,minf(1,y)=0 ;

(3) Along $x=\sqrt{y}$, $f(\sqrt{y},y)=-2y^2-2y$, $0<=y<=1$ $fy(\sqrt{y},y)=-4y-2=0$, $y=-1/2<0$;

F(0,0)=0, f(1,1)=-2-2=-4 , so $maxf(\sqrt{y},y)=0$, $minf(\sqrt{y},y)=-4$.

Summary, max=0 ,min=-4.

讲评：二元函数闭区域最值求，（1）解 $f_x=0$ and $f_y=0$,得到关键点，如果关键在区域内，则求出该点函数值。不在区域内，就丢弃不用考虑。

（2）分别求沿各边界线上的最值，求某边界最值时，就把边界线方程代入函数，函数变为一元函数，再求导，计算极值点，代入算出极值。

（3）比较以上所求函数极值，最大的为最大值，最小的为最小值。